point group give the number of the second order piezomagnetic coefficients required by the magnetic symmetry group induced by that alternating representation of the point group, while the numbers (not in brackets) against a point group indicate the number of the second order coefficients appropriate to the point group. The 21 magnetic symmetry groups, which are induced by the 21 alternating representations of the 32 point groups in each of which the centre of inversion has the character $-1$, and which do not require piezomagnetic coefficients of any order, are not included in the above list.

It is interesting to note that the number of the piezomagnetic coefficients appearing against the equivalent alternating representations (Krishnamurty & Gopalakrishnamurty, 1969) of a point group will be the same. However, from the equality of the numbers of the piezomagnetic coefficients coming under the alternating representations of a point group one cannot conclude that the representations are equivalent. For instance, from the above list one observes that the alternating representations $B_1, A_2$ of the point group $2mm$; $B_1, B_2$ of $\bar{4}2m$ and $A_2'', A_1''$ of $\bar{6}m2$ are not equivalent.

Further one may also notice that the crystallographic point groups $\bar{4}3m$, 432 and $m3m$ require non-vanishing second order piezomagnetic coefficients whereas no first order constants survive for the three point groups (Bhagavantam & Pantulu, 1964; Koptsik, 1966). The appearance of the second order piezomagnetic coefficients for these three cubic point groups will give rise to a greater number of magnetic structures.

From the very structure of the reduced form of the representation $V[[V^2]^2]$, it is evident that an isotropic solid $R^i_\infty$ does not exhibit piezomagnetism since the term $D_0$ (Jahn, 1949) is absent in the reduced form of the representation.

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# Magnetic Symmetry and Limiting Groups

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Using the method of construction of the magnetic symmetry groups already developed by the authors, the magnetic symmetry groups associated with the limiting groups have been derived. The number of constants required to describe the three magnetic properties studied for each one of the derived magnetic symmetry groups is also enumerated.

The crystallographic point groups consist of rotations and rotation-reflexions. Fivefold and higher than sixfold rotation axes are forbidden in the 32 conventional point groups. There is however, a special category of point groups in which infinite-fold rotation axes and reflexions are also permitted symmetry operations. These special groups, seven in number, are called limiting groups (Shubnikov & Belov, 1964), also known as Curie groups. Polycrystalline bodies, fibrous materials like wood, *etc.* belong to a category of substances known as textures. Among a multitude of textures, those possessing the symmetry of limiting groups are of particular interest.

Following the method of the authors (Krishnamurty & Gopalakrishnamurty, 1969) for the construction of the magnetic symmetry groups corresponding to a point group from its real one-dimensional irreducible representations, the magnetic variants of the limiting groups are derived in this note. The numbers of the non-vanishing independent constants in respect of the magnetic properties: (1) pyromagnetism, (2) magnetoelectric polarizability and (3) piezomagnetism, of such

Table 1. *Reduced form of the representation of vectors*

| No. | Representation | Magnetic property known |
|---|---|---|
| 1. $V$ | $D_1^g$ | Pyromagnetism |
| 2. $VV_1$ | $D_0^u + D_1^u + D_2^u$ | Magneto-electric polarizability |
| 3. $V[V^2]$ | $2D_1^g + D_2^g + D_3^g$ | Piezomagnetism |
| 4. $V[[V^2]^2]$ | $4D_1^g + 2D_2^g + 3D_3^g + D_4^g + D_5^g$ | Piezomagnetic coefficients |

Here $D_L^g$ (or $D_L^u$) is a representation of dimensions $2L+1$ of the group $R^i_\infty$ according to its being even (or odd) with respect to inversion

Table 2. *Number of constants for magnetic properties listed in Table* 1.

| *No. / Magnetic symmetry Curie groups | $\infty(A)$ | $\infty m(A_1)$ | $\infty m(A_2)$ | $\infty/m(A')$ | $\infty/m(A'')$ | $\infty 2(A_1)$ | $\infty 2(A_2)$ | $m\infty/m(A_1')$ | $m\infty/m(A_2')$ | $m\infty/m(A_1'')$ | $m\infty/m(A_2'')$ | $\infty\infty(A)$ | $\infty\infty m(A_{1g})$ | $\infty\infty m(A_{1u})$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1. | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 2. | 3 | 1 | 2 | 0 | 3 | 2 | 1 | 0 | 0 | 2 | 1 | 1 | 0 | 1 |
| 3. | 4 | 1 | 3 | 4 | 0 | 1 | 3 | 1 | 3 | 0 | 0 | 0 | 0 | 0 |
| 4. | 11 | 3 | 8 | 11 | 0 | 3 | 8 | 3 | 8 | 0 | 0 | 0 | 0 | 0 |

* Numbers in this column refer to the physical properties given in Table 1, in order.

textures belonging to the symmetry of the limiting groups are also worked out here for the 14 magnetic symmetry groups associated with the Curie groups. These results can also be deduced from the method described by Koptsik (1966).

The seven limiting groups are $\infty$, $\infty\, m$, $\infty/m$, $\infty\, 2$, $m\,\infty/m$, $\infty\,\infty$ and $\infty\,\infty\, m$. It has already been shown by the authors (Krishnamurty & Gopalakrishnamurty, 1969) that the real one-dimensional irreducible representations of a point group $G$ induce the magnetic symmetry groups associated with $G$. When $G$ is taken as one of the seven limiting groups, the identity representation of $G$ induces a magnetic symmetry group, which cannot be distinguished from $G$. The alternating representations of $G$, if any,i nduce the magnetic variants of $G$.

Hence one can list the magnetic variants of the Curie groups from their alternating representations. It may be mentioned that the question of equivalence of the alternating representations does not raise since on two symmetry operations belonging to any two different conjugate classes of the same order of a limiting group are isomorphous (equivalent). Thus the 7 magnetic variants of the Curie groups are described below in terms of the inducing alternating representations* (Tisza, 1933) of the groups:

$$\infty\, m{:}A_2;\ \infty/m{:}A'';\ \infty\, 2{:}A_2;$$
$$m\,\infty/m{:}A_2',\ A_1'',\ A_2'' \text{ and } \infty\,\infty\, m{:}A_{1u}.$$

The groups $\infty$ and $\infty\,\infty$ are not included here as they have no magnetic variants.

If $V$ denotes the representation of an axial vector and $V_1$ that of a polar vector, and $[V^2]$ stands for the symmetrical product of $V$ with itself, one can apply Jahn's (1949) method of reduction of a representation to enumerate the number of the non-vanishing independent constants for the three magnetic properties listed in Table 1. The appropriate form of the representation corresponding to each one of the magnetic properties is given in Table 1.

Using the reductions (Jahn, 1949) of $D_L^u$ and $D_L^g$ for the groups $\infty\, m$, $m\,\infty/m$ and $\infty\, 2$, the number of constants in respect of each one of the magnetic properties is simply given by the numerical coefficients of the inducing real one-dimensional irreducible representations of the Curie groups, in the reduced form of the representation appropriate to the magnetic property under consideration. The results so obtained are summarized in Table 2.

These numbers can also be obtained from the character method (Bhagavantam & Suryanarayana, 1949) by integrating the derived character appropriate to the magnetic property (Rahman, 1953).

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* The notation adopted for the description of the one-dimensional representations of the point groups has already been described by the authors (Krishnamurty & Gopalakrishnamurty, 1969).

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